

Realist model approach to quantum mechanics

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Abstract

The paper proves that quantum mechanics is compatible with the constructive realism of modern philosophy of science. The proof is based on the observation that properties of quantum systems that are uniquely determined by their preparations can be assumed objective without the difficulties that are encountered by the same assumption about values of observables. The resulting realist interpretation of quantum mechanics is made rigorous by studying the space of quantum states—the convex set of state operators. Prepared states are classified according to their statistical structure into indecomposable and decomposable instead of pure and mixed. Simple objective properties are defined and showed to form a Boolean lattice.

1 Introduction

If our aim is to propose a realist interpretation of quantum mechanics, we ought to ask first, what is realism. The abstract realist doctrine, "The world is not just a construction of our mind, but does really exist", cannot be falsified because it is vague, but then, of course, not of much use. The so-called naive realism, "The real world is as we perceive it", is clearly false. The metaphysical realism [1], "There is exactly one true and complete description of 'the way the world is'", is surely near to a physicist's heart, but it seems to be rather different from what the history of physics teaches us.

In the contemporary philosophy of science, there is a stream that can be characterised by the doctrine "Our knowledge is dominated by a family of models". It was founded and is represented eg by Patrick Suppes, Bas van Fraassen, Ronald Giere, Wolfgang Stegmüller and Nancy Cartwright. Very schematically, it can be described as follows. Each grand theory of physics (such as Newtonian mechanics or quantum mechanics) can be divided into two parts:

- Treasure of successful models. The set is open in the sense that it grows with time.
- General language part, which contains mathematical structure of state space, of dynamical equations, of symmetries and the form of observables. It is considered as secondary, because it is only obtained through study of models, or serves as an instrument of model construction as well as of unification and classification of different models.

A particular philosophy within the stream is the so-called Constructive Realism by R. N. Giere [2], which can be roughly characterised as follows:

- Each model is constructed with the aim to give an approximative representation of some aspects of a real object.
- Concepts of the general language part are connected to the reality only via models.

It is a relatively strong kind of realism, and it seems to correspond very well to how physics works in practice. The present paper follows [3], where this philosophy is accepted and its application to quantum mechanics is described in more details.

In particular, each model has two components: an empirical one being an identifiable real object, and a language one, being a theoretical construct that models the object.

Our work on quantum mechanics [4, 5, 6, 7, 8, 9, 10, 3] is aimed at constructing quantum models of classical world. Within the constructive realism, the relation

between quantum and Newtonian mechanics, say, ought to be studied by comparing quantum and Newtonian models of one and the same real object rather than looking for a relation between their general language parts.

To prevent misunderstandings, let us stress that we are not trying to construct any universal map between states of quantum mechanics and those of Newtonian mechanics, such as Wigner-Weyl-Moyal map (see eg [11], p 85). Our aim is to construct quantum models of classical world, not vice versa. We start by distinguishing three things: a real object, its Newtonian and its quantum model. Then, only such real objects are considered that are macroscopic and possess successful classical Newtonian models. Thus, a hydrogen atom does not qualify. (It can be a real object within our interpretation.) Moreover, the quantum model will, in most cases, be much richer, than the classical one. For instance, properties of a Newtonian model of a free real macroscopic solid body are eg isotropic continuum distribution of matter with a given mass density, elastic coefficient, total momentum and angular momentum. Properties of a quantum model can be eg all particle numbers that form the body, their masses and spins, their Hamiltonian with an interaction potential and a suitable quantum state of the whole system.

2 An heuristic principle of quantum realism

The project runs into three well-known problems:

1. Classical properties are objective and robust, see [12]. How can they emerge from quantum mechanics, which does not seem to allow such properties?
2. Realism leads quite generally to contradiction with the linear quantum evolution in the measurement theory (Schrödinger cat, see [12]).
3. What are real quantum objects to construct models of? It seems correct to require that language parts of models ascribe enough objective properties to their empirical parts. However, there is an apparent lack of objective properties in the language part of quantum models. This makes implausible to assume the reality of quantum systems, see [13].

The problems ought to be dealt with simultaneously (see [3]), but we shall focus just on the third problem here.

We define objective properties as follows. Let \mathcal{S} be a real object and \mathcal{S}_m its theoretical model. \mathcal{S}_m contains two kinds of information. First, it describes \mathcal{S}_m in sufficient detail by values of some quantities that can be ascribed to \mathcal{S} alone independently of any observations. Second, it enables to calculate what will be observed on \mathcal{S} if some measurement is carried out. An objective property of \mathcal{S} is

the value of a quantity of the first kind. Thus, the values of observables of quantum system \mathcal{S} are *not* objective properties of \mathcal{S} . To ascribe values of observables to \mathcal{S} leads to contradictions such as contextuality (for more detail, see [3]). Here, we accept the old tenet: each such value is only created during registration of \mathcal{S} by some apparatus \mathcal{A} and it is an objective property of the composite $\mathcal{S} + \mathcal{A}$ rather than of \mathcal{S} alone. It gives only an indirect information about \mathcal{S} .

Fortunately, there are other observable properties of quantum systems. They can be described by the following heuristic principle [4].

Basic Ontological Hypothesis of QM *A property is objective if its value is uniquely determined by a preparation according to the rules of standard QM. The "value" is the value of the mathematical expression that describes the property and it may be more sophisticated than just a real number. To observe an objective property, many registrations of one or more observables are necessary.*

In fact, the Hypothesis just states explicitly the meaning that is tacitly given to preparation by standard quantum mechanics. More discussion on the meaning of preparation is in [6, 9]. In any case, prepared properties can be ascribed to the prepared system without any problems or paradoxes. Often, we shall use the Hypothesis as an heuristic principle: it will just help to find some specific properties and then it will be forgotten, that is, an independent assumption will be made that these properties can be objective and each of them will be further studied.

It is easy to give examples of such objective properties. We can classify them into *structural* and *dynamical*. The structural ones are eg mass, charge, spin, structure of Hamiltonian of an isolated system, etc The dynamical ones are eg state operator, average value and variance of an observable, etc

Often, the Hypothesis meets one of the following two questions. First, how can it be applied to cosmology, when there was nobody there at the Big Bang to perform any state preparation? Second, a state preparation is an action of some human subject; how can it result in an objective property? Both objections result from the view that preparations are just some manipulations in human laboratory. This is too narrow: even textbooks (see eg [13]) notice that preparations can also run spontaneously as natural processes, such as the preparations of neutrinos that arrive at the Earth from the core of the Sun. Moreover, the second objection is not much more than a pun. It is not logically impossible that a human manipulation of a system results in an objective property of the system. For example, pushing a snooker ball imparts it a certain momentum and angular momentum that can be assumed to be objective properties of the pushed ball.

3 The space of quantum states

Quantum states of objects that we meet in our everyday life are very different from wave functions. Thus, the usual focus on wave functions is misleading if we want to understand classical properties [5]. We have to work with general states, which are described by what is usually called "density matrices" [13] or "state operators" [14].

3.1 Mathematical preliminaries

Let us collect some mathematics on quantum states that can be found in literature but is not generally known.

A bounded self-adjoint operator \mathbf{a} is called *positive* if

$$\langle \psi | \mathbf{a} | \psi \rangle \geq 0 \quad \forall |\psi\rangle \in \mathbf{H}$$

and *trace-class* if

$$\text{tr}[\sqrt{\mathbf{a}^2}] < \infty .$$

The left-hand side of the inequality can be shown to be a norm and the space of trace-class operators completed with respect to this norm is a Banach space. With the partial ordering $\mathbf{a} > \mathbf{b}$ defined by operator $\mathbf{a} - \mathbf{b}$ being positive, it is an ordered Banach space. (For more details, see [15], Appendix III and IV.)

What is the true state space of quantum mechanics?

Definition 1 Let \mathcal{S} be a quantum system with Hilbert space \mathbf{H} . Let $\mathbf{T}(\mathbf{H})$ denote the ordered Banach space of trace-class operators on \mathbf{H} . State space $\mathbf{T}(\mathbf{H})_1^+$ of \mathcal{S} is the set of all positive elements of $\mathbf{T}(\mathbf{H})$ with trace 1.

Hence, the space of states is the intersection of the unit sphere and the positive cone of the Banach space $\mathbf{T}(\mathbf{H})$. As the positive cone has a non-trivial boundary in $\mathbf{T}(\mathbf{H})$, $\mathbf{T}(\mathbf{H})_1^+$ has a rich face structure.

The space $\mathbf{T}(\mathbf{H})_1^+$ is not a linear space but it is *convex*, that is, invariant with respect to *convex combination*,

$$w\mathbf{T}_1 + (1 - w)\mathbf{T}_2 = \mathbf{T} ,$$

where $\mathbf{T}_{1,2} \in \mathbf{T}(\mathbf{H})_1^+$ and $w \in [0, 1]$. The states \mathbf{T}_1 and \mathbf{T}_2 are called *convex components* of \mathbf{T} .

Definition 2 Face \mathbf{F} is a norm-closed subset of $\mathbf{T}(\mathbf{H})_1^+$ that is invariant with respect to convex combinations and contain all convex components of any $\mathbf{T} \in \mathbf{F}$.

"Face" is an important notion of the mathematical theory of convex sets.

Theorem 1 Every face $\mathbf{F} \subset \mathbf{T}(\mathbf{H})_1^+$ can be written as $\mathbf{F}(\mathbf{T})$ for a suitably chosen $\mathbf{T} \in \mathbf{F}(\mathbf{T})$ where $\mathbf{F}(\mathbf{T})$ is the smallest face which contains \mathbf{T} .

For proof, see [15], p 76. Thus, each state operator lies in some face. There is a useful relation between faces and projections:

Theorem 2 To each face \mathbf{F} of $\mathbf{T}(\mathbf{H})_1^+$ there is a unique projection $\mathbf{P} : \mathbf{H} \mapsto \mathbf{H}'$, where \mathbf{H}' is a closed subspace of \mathbf{H} , for which $\mathbf{T} \subset \mathbf{F}$ is equivalent to

$$\mathbf{T} = \mathbf{P}\mathbf{T}\mathbf{P} .$$

The map so defined between the set of faces and the set of projections is an order isomorphism ie it is invertible and $\mathbf{P}_1 < \mathbf{P}_2$ is equivalent to $\mathbf{F}_1 \subset \mathbf{F}_2$.

For proof, see [15], p 77. We shall denote the face that corresponds to a projection \mathbf{P} by $\mathbf{F}_\mathbf{P}$.

Clearly, intersection of two faces, if non-empty, is a face and a unitary map of a face is a face. The next theorem shows that $\mathbf{F}(\mathbf{T})$ is not necessarily the set of all convex components of \mathbf{T} .

Theorem 3 Let $\mathbf{P}(\mathbf{H})$ be infinite-dimensional. Let $\mathbf{T}_1, \mathbf{T}_2 \in \mathbf{F}_\mathbf{P}$ and be positive definite on $\mathbf{P}(\mathbf{H})$. Then

$$\mathbf{F}(\mathbf{T}_1) = \mathbf{F}(\mathbf{T}_2) = \mathbf{F}_\mathbf{P} .$$

Let $\{|k\rangle\}$ be an orthonormal basis of $\mathbf{P}(\mathbf{H})$ and let

$$\sup_k \frac{\langle k|\mathbf{T}_1|k\rangle}{\langle k|\mathbf{T}_2|k\rangle} = \infty . \quad (1)$$

Then \mathbf{T}_1 is not a convex component of \mathbf{T}_2 .

For proof, see [3].

Definition 3 $\mathbf{T} \in \mathbf{T}(\mathbf{H})_1^+$ is called extremal if it lies in a zero-dimensional face.

For extremal states, we have:

Theorem 4 \mathbf{T} is extremal iff $\mathbf{T} = |\psi\rangle\langle\psi|$, where ψ is a unit vector of \mathbf{H} .

For proof, see [15], p 78. Thus, extremal state can be described by elements of \mathbf{H} , and only the q -representation of such an element is a "wave function".

3.2 Physical interpretation of the mathematics

Here, the mathematical consequences of our heuristic principle will be worked out. The physical meaning of elements of $\mathbf{T}(\mathbf{H})_1^+$ is given by

Assumption 1 *Let \mathcal{S} be a quantum system with Hilbert space \mathbf{H} . Every $\mathsf{T} \in \mathbf{T}(\mathbf{H})_1^+$ can be prepared as a state of \mathcal{S} and is then an objective property of \mathcal{S} .*

Next, we start to give a possible physical meaning to convex combinations.

Definition 4 *Let \mathcal{P}_1 and \mathcal{P}_2 be two preparations of \mathcal{S} and $w \in [0, 1]$. Statistical mixture,*

$$\mathcal{P} = \{(w, \mathcal{P}_1), ((1 - w), \mathcal{P}_2)\} , \quad (2)$$

of \mathcal{P}_1 and \mathcal{P}_2 is the following preparation: Let \mathcal{S} be prepared either by \mathcal{P}_1 or by \mathcal{P}_2 in a random way so that \mathcal{P}_1 is used with probability w and \mathcal{P}_2 with probability $1 - w$.

Assumption 2 *Let \mathcal{P}_1 and \mathcal{P}_2 be two preparations of \mathcal{S} and let them prepare states T_1 and T_2 . Then the statistical mixture (2) prepares state*

$$\mathsf{T} = w\mathsf{T}_1 (+)_p (1 - w)\mathsf{T}_2 .$$

We call the right-hand side statistical decomposition of T .

The purpose of sign “ $(+)_p$ ” on the right-hand side is to stress that this convex combination is a statistical decomposition. To be aware of the distinction is very important for the understanding of the theory of quantum measurement, see [3]. For example, the theory of quantum decoherence can achieve that the final state of the apparatus is a convex combination of distinct pointer states but cannot conclude that it is a statistical decomposition and must, therefore, resort to further assumptions such as Everett interpretation [16]. Let us stress that a statistical decomposition of state T is not determined by the mathematical structure of state operator T but by a preparation of T .

Sometimes, one meets the objection that states $w\mathsf{T}_1 (+)_p (1 - w)\mathsf{T}_2$ and $w\mathsf{T}_1 + (1 - w)\mathsf{T}_2$ of system \mathcal{S} cannot be distinguished by any measurement. But this is only true if the measurements are limited to registrations of observables of \mathcal{S} . If observables of arbitrary composite systems containing \mathcal{S} are also admitted, then the difference between a statistical decomposition and a convex combination can be found by measurements [7]. This is exactly the argument against the decoherence theory described in [17], p 171. Let us also emphasise that quantum state statistics has nothing to do with the statistics of values of observables.

As a mathematical operation, $(+)_p$ is commutative and associative. Thus, the definitions and assumptions can be generalised to more than two preparations and states. Moreover, state decomposition is invariant with respect to system composition and unitary evolution [6].

Definition 5 *Prepared state \mathbf{T} of the form $w\mathbf{T}_1 (+)_p (1-w)\mathbf{T}_2$ with $w \in (0,1)$ is called decomposable.*

The meaning of a prepared state \mathbf{T} being indecomposable is that the ensemble \mathbf{E} defined by repeating the preparation \mathcal{P} of \mathbf{T} has no sub-ensemble \mathbf{E}' that can be obtained by repeating preparation \mathcal{P}' of state $\mathbf{T}' \neq \mathbf{T}$.

Thus, to be decomposable or indecomposable are *physical* properties of prepared states that are determined by the preparations of the states, not just by *mathematical* existence of convex combinations. The faces just restrict possible statistical decomposition of states. For example, any extremal state is *indecomposable*. Decomposable states has been also called "proper mixtures" [17], "direct mixtures" [15] or "gemenges" [14, 7].

There are prepared non-extremal states that are indecomposable. Consider system \mathcal{S} prepared in the EPR experiment (see eg [13], p 150). \mathcal{S} is a spin 0 system composite of two spin 1/2 systems \mathcal{S}_1 and \mathcal{S}_2 . \mathcal{S} is prepared in extremal state $|\psi\rangle\langle\psi|$,

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|1+\rangle \otimes |2-\rangle - |1-\rangle \otimes |2+\rangle) ,$$

where $|1+\rangle$ is the state of the first particle with spin up etc, so that the z components of the spins of the two subsystems are anti-correlated. Then the state of \mathcal{S}_1 is

$$tr_{\mathcal{S}_2}[|\psi\rangle\langle\psi|] = \frac{1}{2}(|1+\rangle\langle 1+| + |1-\rangle\langle 1-|) , \quad (3)$$

which is not extremal but the right-hand side on Eq. (3) cannot be a statistical decomposition because this would imply that Bell inequality were satisfied. Here, $tr_{\mathcal{S}_2}[|\psi\rangle\langle\psi|]$ is a partial trace of $|\psi\rangle\langle\psi|$ over the degrees of freedom of \mathcal{S}_2 , see eg [13].

By the way, the experiment can also be considered as a preparation of \mathcal{S}_1 in state (3). This state is then an objective property of \mathcal{S}_1 irrespectively of the entanglement of \mathcal{S}_1 with \mathcal{S}_2 . One can also say that $|\psi\rangle\langle\psi|$ is an entangled state,

$$|\psi\rangle\langle\psi| \neq tr_{\mathcal{S}_2}[|\psi\rangle\langle\psi|] \otimes tr_{\mathcal{S}_1}[|\psi\rangle\langle\psi|] ,$$

but (3) is not.

We observe that a quantum state is conceptually very different from a state in Newtonian mechanics. It may be helpful to look at some important differences. Let us define a state of a Newtonian system as point p of the phase space of the system, Γ . Newtonian state defined in this way is generally assumed to satisfy:

1. *Objectivity*: a state is an objective property.
2. *Universality*: any system is always in some state.

3. *Exclusivity*: a system cannot be in two different states simultaneously.
4. *Completeness*: any state of an isolated system represents the maximum information that can exist about this system.
5. *Locality*: the state of a systems determines the position of the system.

An incomplete information about the state of a system can be described by a probability distributions on $\mathbf{\Gamma}$. Such a distribution is sometimes called *statistical state*. In any case, we distinguish a state from a statistical state.

A quantum state is an element of $\mathbf{T}(\mathbf{H})_1^+$ and the comparison to Newtonian states is described by:

1. There is objectivity: a prepared state is an objective property.
2. There is no universality: a system need not be in any state (an example is a particle \mathcal{S} in a system \mathcal{S}' of identical particles and we assume that a state of \mathcal{S}' has been prepared but that of \mathcal{S} has not [9]).
3. There is no exclusivity: a system can be in several states simultaneously (such as \mathbf{T}_1 and $w\mathbf{T}_1 (+)_p (1 - w)\mathbf{T}_2$ above).
4. There is no completeness: a state operator alone does not contain any information on the statistical decomposition of the prepared state. However, if an indecomposable state of an isolated system is given, no more knowledge on the system can objectively exist than that given by the state.
5. Quantum states are non-local: most states of a single particle do not determine its position, but two detectors at different positions will give anti-correlated results ([9]).

In particular, we have the following correspondences:

$$\begin{aligned} \text{quantum indecomposable state} &\longleftrightarrow \text{Newtonian state} \\ \text{quantum decomposable state} &\longleftrightarrow \text{Newtonian statistical state} \end{aligned}$$

There is, of course, the difference that a decomposable state is described by an element of $\mathbf{T}(\mathbf{H})_1^+$, whereas a Newtonian statistical state is not an element of $\mathbf{\Gamma}$. From the point of view of both statistics of states and statistics of values of observables, textbooks focus on the difference between *pure* (extremal) and *mixed* (non-extremal) states is rather misleading.

An important consequence of our interpretation is the following. We consider any indecomposable state as a complete description of the reality of the system. Thus, the collapse of wave function and analogous processes must be viewed as physical processes, not just as changes of our information about the system [6, 9, 3].

3.3 Simple objective properties

In quantum mechanics, simple objective properties can be defined as follows.

Definition 6 *Let $f : \mathbf{T}(\mathbf{H})_1^+ \mapsto \mathbb{R}$. A simple objective property is defined by proposition " f has value a ".*

As states are uniquely determined by preparations, and values of f by the states, the simple properties are objective. Each simple property is equivalent to a subset of $\mathbf{T}(\mathbf{H})_1^+$ described by

$$\{\mathbf{T} \in \mathbf{T}(\mathbf{H})_1^+ \mid f(\mathbf{T}) = a\} .$$

Quantum simple properties form a Boolean lattice with respect to logical operations of union and intersection that is isomorph to the lattice of subsets of $\mathbf{T}(\mathbf{H})_1^+$. This is clearly different from the so-called "quantum logic", which holds for properties defined by propositions "Projection \mathbf{P} has value η " with $\eta = 0, 1$ (see eg [18]). The reason is that our simple properties are defined by preparations while those of quantum logic by registrations because \mathbf{P} is an observable and η its value.

Quantum simple properties are analogous to Newtonian properties that can be defined as real functions on $\mathbf{\Gamma}$. For example, a system having energy E is a property equivalent to the subset of points of $\mathbf{\Gamma}$ at which the energy has value E . Newtonian properties also form a Boolean lattice.

A very important example of quantum simple property is average $tr[\mathbf{A}\mathbf{T}]$ of observable \mathbf{A} in state \mathbf{T} . One could ask how comes that average of \mathbf{A} is an objective property while its values are not? But this is simple: the average can be considered as a predetermined condition on the values that are to be registered: in large numbers these values must add to the average. Thus, the average is an example of a property that can be measured by a large number of individual registrations.

Clearly there are enough simple objective properties to describe the dynamical situation of any quantum object completely. Thus, our theoretical models of real objects can ascribe them enough objective properties, as required. More discussion is in [4, 7, 3].

4 Conclusion and outlook

- Constructive Realism by Giere—a kind of relatively strong realism—is shown to be compatible with quantum mechanics.
- The resulting realist interpretation is indispensable for our method to construct quantum models of classical properties [5, 3] and of measurement processes [6, 8, 3].

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